



MATHCAD Fundamentals and Functions – Session II

EGN 1006 – Introduction to Engineering



MATHCAD

Matrix Operations and Graphing



Indexing

You can use an integer range variable to create and indexed array. The array **MUST** be indexed by the integer range variable so that the correct elements are filled.

Enter: **[i][:][0][;][10]**

[time_t](i – placed as the subscript)[:]*i*/20

[i][:][time*10]

[i][=]

[time][=]



Making changes to the index

Enter

```
[x][:][time*20]
```

```
[x][=]
```

Now change i in the time equation to i^2 and notice the changes.

Indexing

```
i := 0..10
```

```
timei :=  $\frac{i}{20}$ 
```

```
x := time·10
```

i =

	0
0	0
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8
9	9
10	10

time =

	0
0	0
1	0.05
2	0.1
3	0.15
4	0.2
5	0.25
6	0.3
7	0.35
8	0.4
9	0.45
10	0.5

x =

	0
0	0
1	$5 \cdot 10^{-3}$
2	0.01
3	0.015
4	0.02
5	0.025
6	0.03
7	0.035
8	0.04
9	0.045
10	0.05



Indexing Summary

The variable "i" is simply a *range variable* while "time" and "x" are called *one-dimensional arrays* or *matrices* indexed with "i" .

- Notice that the initial default index or "origin" for the array is automatically set to ZERO.
- The "origin" may be changed by re-assigning a value to a pre-defined constant



Changing the Origin

Enter:

[ORIGIN][:][1]

Place this at top of worksheet

Notice what happens!

Now DRAG the box so it is UNDER the time equation.

Now DRAG the box so it is UNDER all of the worksheet. You should notice that the values for the origin change on the data table



Querying the array

If you have a ton of values in the array and you want to know, for example, ***what is the time at 3?***

Enter:

[time₃]=

Note: Remember that to get the INDEX subscript you use the LEFT bracket "[" not the PERIOD which is used for a TEXT SUBSCRIPT.



Matrix Definitions

Multi-dimensional arrays can be defined by specifying multiple indices separated by commas.

Enter the following:

```
[i][:][0][;][10]
```

```
[j][:][0][;][10]
```

```
[Ai,j][:][(i2+j3)/4]
```

```
[A][=]
```

Then evaluate: [A_{2,3}][=]

Note: In this example "i" was the ROW and "j" was the column

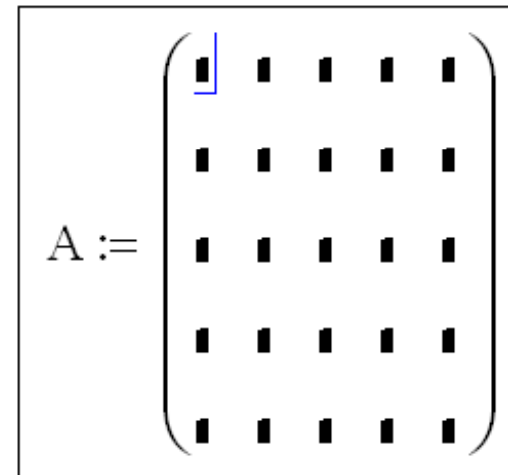
Matrix Definitions

A matrix or array can also be directly defined by specifying its dimensions and entries using the *insert-matrix* or the **<Ctrl-M>** shortcut.

Enter the following:

[A][:][<Ctrl-M>]

Specify the # of columns and rows and enter the elements of the new matrix



The diagram shows the definition of a matrix A. On the left, the text "A := " is followed by a large right-facing curly parenthesis. Inside this parenthesis, there is a 5x5 grid of small black squares representing matrix elements. A blue vertical line with a downward-pointing arrow is positioned to the left of the top-left element, indicating the starting point for entering the matrix dimensions.



Reading Data Files

- Files may be imported into MATHCAD in the form of matrices by using the following command

A:=READPRN("Filename.txt")

- Any extension can be specified
- You must provide the full path name if the file is not in the MATHCAD directory
- After execution of this command, the ordered values on the filename.txt will be imported into A in the form of an $A_{i,j}$ ordered matrix



Entering Data Files

Enter the following:

[M][:][READPRN("c:\desktop\UCF\data.txt")]

[M][=]

To query enter

[M_{3,4}][=] NOTE: Use the left bracket to subscript down



Writing Data Files

A matrix can be exported into a data file by using the following command:

WRITEPRN("Filename.txt"):=A

- Any extension can be specified
- The data file will be written in the directory MATCAD is in
- After the execution of the command the ordered values of matrix A will then be exported to the data file named filename.txt



Writing Data Files

Enter the following

```
[D][:][M][*][2]
```

```
[D][=]
```

```
[WRITEPRN][("c:/desktop/UCF/data2.  
txt")][:][D]
```

Then go to your desktop, open up the UCF folder and check to see if a file called data2.txt was created. Notice the time and date of creation.



Editing the data file

Suppose the file contains values that need to be adjusted for accuracy and precision. Enter this above the WRITEPRN command:

**[PRNPRECISION][:][2]
[COLUMNPRECISION][:][2]**

Then click inside the WRITE command box and hit ENTER. This will RE-SAVE the data file with the adjusted values. Open the file to see and verify the new changes.



Matrix Operations

A matrix can be operated over using built in MATHCAD functions.

- Adding or subtracting

$$C:=A+B$$

- Multiplying by a scalar

$$C:=h*A$$

- To access a column in a matrix use <Ctrl-6>

$$V:=A^{<3>} \text{ where } V = \text{vector}$$



Matrix Operations

- Transpose of a matrix using <Ctrl-1>

$$G := F^T$$

- Matrix Multiplication is UNITS are CONSISTANT!!!

$$C := A * B$$

- Determinant (for square matrices) using <shift-\>
Note: "Square" means same # of rows as columns

$$D := |A|$$

- Inverse (for square matrices) using $\wedge -1$

$$D := A^{-1}$$



Matrix Operations

Do the following:

Display matrix A from earlier in the lesson

Create another FIVE by FIVE matrix and call it B

Enter the values then enter

```
[C][:][A][+][B]
```

```
[C][=]
```



Matrix Operations

Enter the following:

[D][:][1.4][*][A]

[D][=]

[E][:][A^T] –using Ctrl-1

[V][:][E^{<0>}] – using Ctrl-6



Matrix Operations- The determinant

What are we doing?

The beginnings of matrices and determinants goes back to the second century BC although traces can be seen back to the fourth century BC. However it was not until near the end of the 17th Century that the ideas reappeared and development really got underway.



Matrix Operations- The determinant

It is not surprising that the beginnings of matrices and determinants should arise through the study of systems of linear equations. The Babylonians studied problems which lead to simultaneous linear equations and some of these are preserved in clay tablets which survive. For example a tablet dating from around 300 BC contains the following problem:-

There are two fields whose total area is 1800 square yards. One produces grain at the rate of $\frac{2}{3}$ of a bushel per square yard while the other produces grain at the rate of $\frac{1}{2}$ a bushel per square yard. If the total yield is 1100 bushels, what is the size of each field.



Matrix Operations- The determinant

The Chinese, between 200 BC and 100 BC, came much closer to matrices than the Babylonians. Indeed it is fair to say that the text [Nine Chapters on the Mathematical Art](#) written during the Han Dynasty gives the first known example of matrix methods. First a problem is set up which is similar to the Babylonian example given above:-

There are three types of corn, of which three bundles of the first, two of the second, and one of the third make 39 measures. Two of the first, three of the second and one of the third make 34 measures. And one of the first, two of the second and three of the third make 26 measures. How many measures of corn are contained of one bundle of each type?



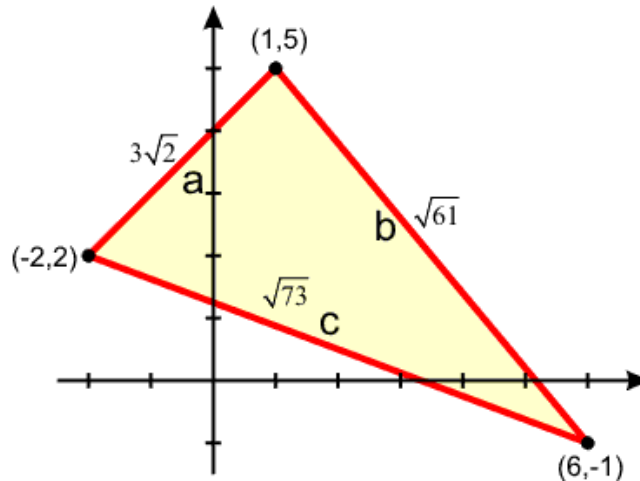
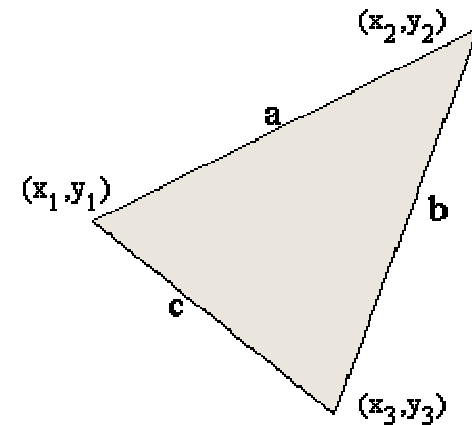
Matrix Operations- The determinant

- Now the author does something quite remarkable. He sets up the coefficients of the system of three linear equations in three unknowns as a table on a 'counting board'.

1	2	3
2	3	2
3	1	1
26	34	39

Determinant

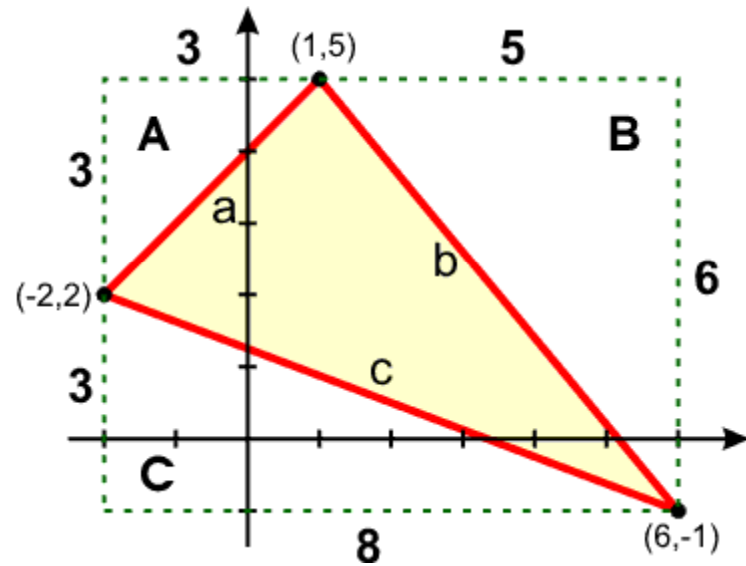
- Consider a triangle with vertices at (x_1, y_1) , (x_2, y_2) , and (x_3, y_3) . If the triangle was a right triangle, it would be pretty easy to compute the area of the triangle by finding one-half the product of the base and the height.
- However, when the triangle is not a right triangle, there are a couple of other ways that the area can be found.



Consider the triangle with vertices at $(-2, 2)$, $(1, 5)$, and $(6, 1)$.

Determinant

- The triangle can be enclosed in a rectangle. The vertices of the triangle will intersect the rectangle in three places, forming three right triangles. These triangles are denoted A, B, and C in the picture.





Determinant- A geometric technique

The area of the triangle we desire will be the area of the rectangle minus the areas of the three triangles.

$$\text{Area of triangle A} = 3 (3) / 2 = 9/2.$$

$$\text{Area of triangle B} = 5 (6) / 2 = 15.$$

$$\text{Area of triangle C} = 8 (3) / 2 = 12.$$

The sum of the areas of the triangles is $9/2 + 15 + 12 = 63 / 2$ or 31.5.

The area of a rectangle is base times height, so the bounding rectangle has area = $8 (6) = 48$.

The area of the triangle in the middle is the difference between the rectangle and the sum of the areas of the three outer triangles.

$$\text{Area of triangle} = 48 - 31.5 = 16.5.$$



Determinant

- It turns out that the area of a triangle can also be found using determinants.
- What you do is form a 3×3 determinant where the first column are the x's for all the points, the second column are the y's for all the points, and the last column is all ones.

	x	y	1
point 1	-2	2	1
point 2	1	5	1
point 3	6	-1	1



Determinant

$$Area_{\Delta} = \frac{1}{2} \left[(x_3y_2 - x_2y_3) - (x_3y_1 - x_1y_3) + (x_2y_1 - x_1y_2) \right]$$

$$(6*5)-(1*-1)=31$$

$$(6*2)-(-2*-1)=10$$

$$(1*2)-(-2*5)=12$$

$$31-10+12=33$$

$$33/2=16.5$$

This is the same value for the area as the geometric technique



What are determinates used for?

- Solving MULTIPLE sets of equations
- The determinant is actually the CROSS PRODUCT in vector physics
- Used to define Integrals in Calculus
- Decryption and coding



Determinant

Insert a page break, then

[A]=

[B]=

Enter the following

[D][:]|A| - using shift \

[D][=]

[E][:]|B| - using shift \

[E][=]

Inverse (for square matrices) using \wedge^{-1}

$F:=A^{-1}$, then $F=$ or $A^{-1}=\$



Other Matrix Functions

Other functions include:

- `matrix(r,c,f)`

- `identity(c)`

Enter `d:=identity(6)` then `d=`

- `submatrix(A,r1,r2,c1,c2)`

Enter `A=`, then `c:=submatrix(A,0,2,0,3)`

`C=`

- `augment(A,B)`

Enter `C:=augment(A,B)` then `C=`

- `stack(A,B)`

Enter `C:=stack(A,B)` then `C=`



Additional Matrix Functions

- `sort(V)` for 1-D arrays(Vectors)
- `reverse(V)`
- `csort(A,c#)`
- `rsort(A,r#)`



Matrix System Solvers (The Bottom Line)

To solve a system of simultaneous linear equations in MATHCAD

$$[A]\{x\} = \{b\}$$

The system may be re-ordered as

$$\{x\} = [A]^{-1}\{b\}$$

Or the function ***Isolve()*** may be used

$$x := \text{Isolve}(A, b)$$



Matrix System Solvers

Add a page break and enter $A=$

Then the determinant, $d:=|A|$ using
shift-\, then $d=.$

Define $b:=(\text{insert-matrix})$ with 5 rows
and 1 column, Enter in 5,7,-8,3,5)

Enter:

$x:=A^{-1}*b$, then $x=$ also

$x:=\text{lsolve}(A,b)$ then $x=$

Matrix System Solvers

$$A = \begin{pmatrix} 9 & -2 & 0 & -4 & 7 \\ 0 & 19 & 5 & -6 & 5 \\ 0 & 0.4 & 6 & 33 & 7 \\ 9 & 7 & 8 & -6 & 8 \\ 52 & 11 & 0 & 6 & 5 \end{pmatrix} \quad b = \begin{pmatrix} 5 \\ 7 \\ -8 \\ 3 \\ 5 \end{pmatrix}$$

Notice that the direct calculation of the inverse of A times b gave you the same solution as the *lsolve* command using A and b.

$$x := A^{-1} \cdot b$$

$$x := \text{lsolve}(A, b)$$

$$x = \begin{pmatrix} -4.287 \times 10^{-3} \\ 0.31 \\ -0.745 \\ -0.252 \\ 0.665 \end{pmatrix} \quad x = \begin{pmatrix} -4.287 \times 10^{-3} \\ 0.31 \\ -0.745 \\ -0.252 \\ 0.665 \end{pmatrix}$$

These are TWO COMPLETELY different ways to solve the equations and YOU WILL get different answers if you have over 200 equations. The *lsolve* command is faster.

+



Plotting Data

Plotting in MATHCAD can be done two different ways:

- Using indexed variables
- Using functions



Plotting with Indexed Variables

Enter the following:

$i := 0..100$

$time_i := (i/100)^2$

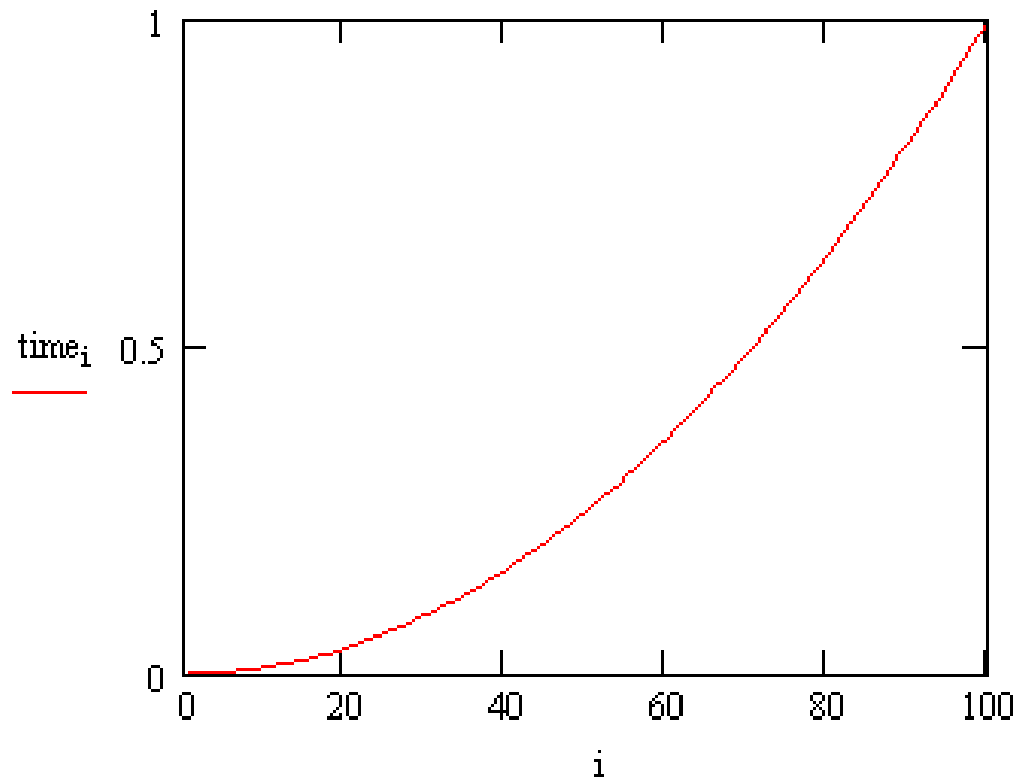
From the insert menu choose **Graph**
then **XY plot**

On the x-axis placeholder type "i"

On the y-axis placeholder type "time_i"

You should see the following

$$i := 0..100 \quad \text{time}_i := \left(\frac{i}{100}\right)^2$$





Plotting with functions

Enter the following:

$y(x) := x^2$

$x := 0, 0.01 \dots 1$

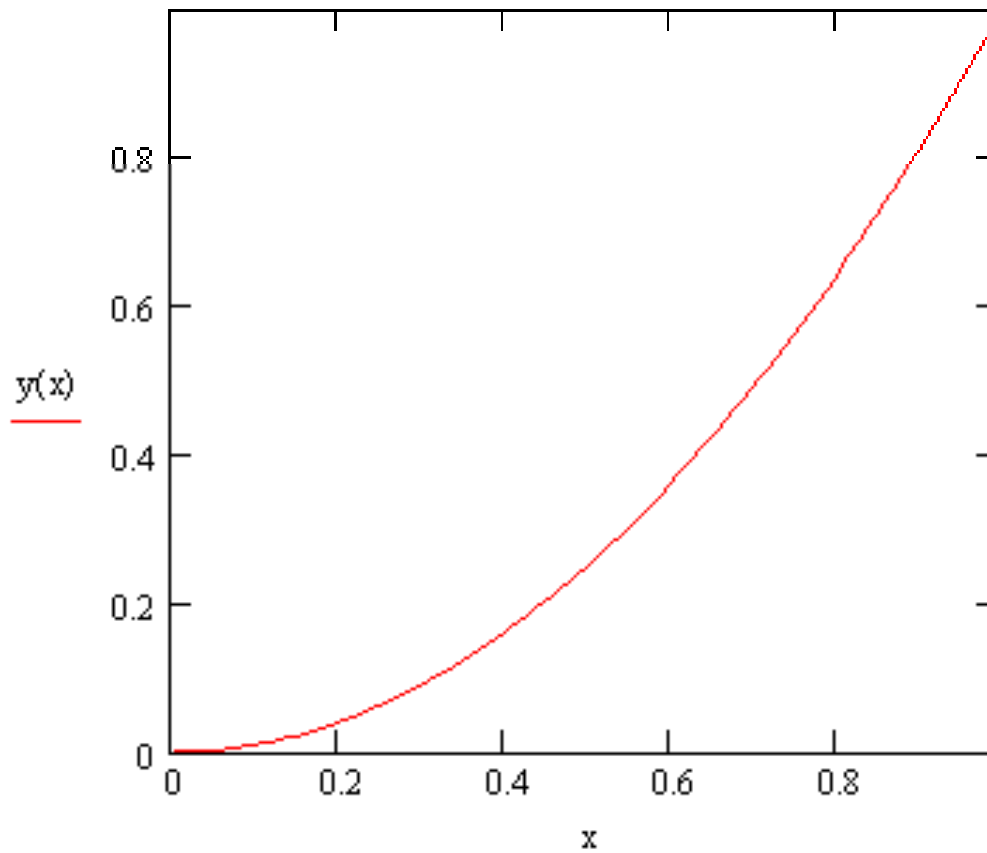
From the insert menu choose **Graph**
then **XY plot**

On the x-axis placeholder type "x"

On the y-axis placeholder type "y(x)"

You should see the following

$$y(x) := x^2 \quad x := 0, 0.01 \dots 1$$





Graph Editing

Double click on the graph

- Turn on the grid lines
- Turn off AUTO GRID and specify how many lines you want on each axis
- Click on trace one and change the color and whether it is a solid line or dashed line
- Or instead of a line you can choose dots and symbols to represent those dots
- You can also change the equation to see an immediate change in the graph
- If you click on the #s on the axis you can change the range of each axis



Enter the following

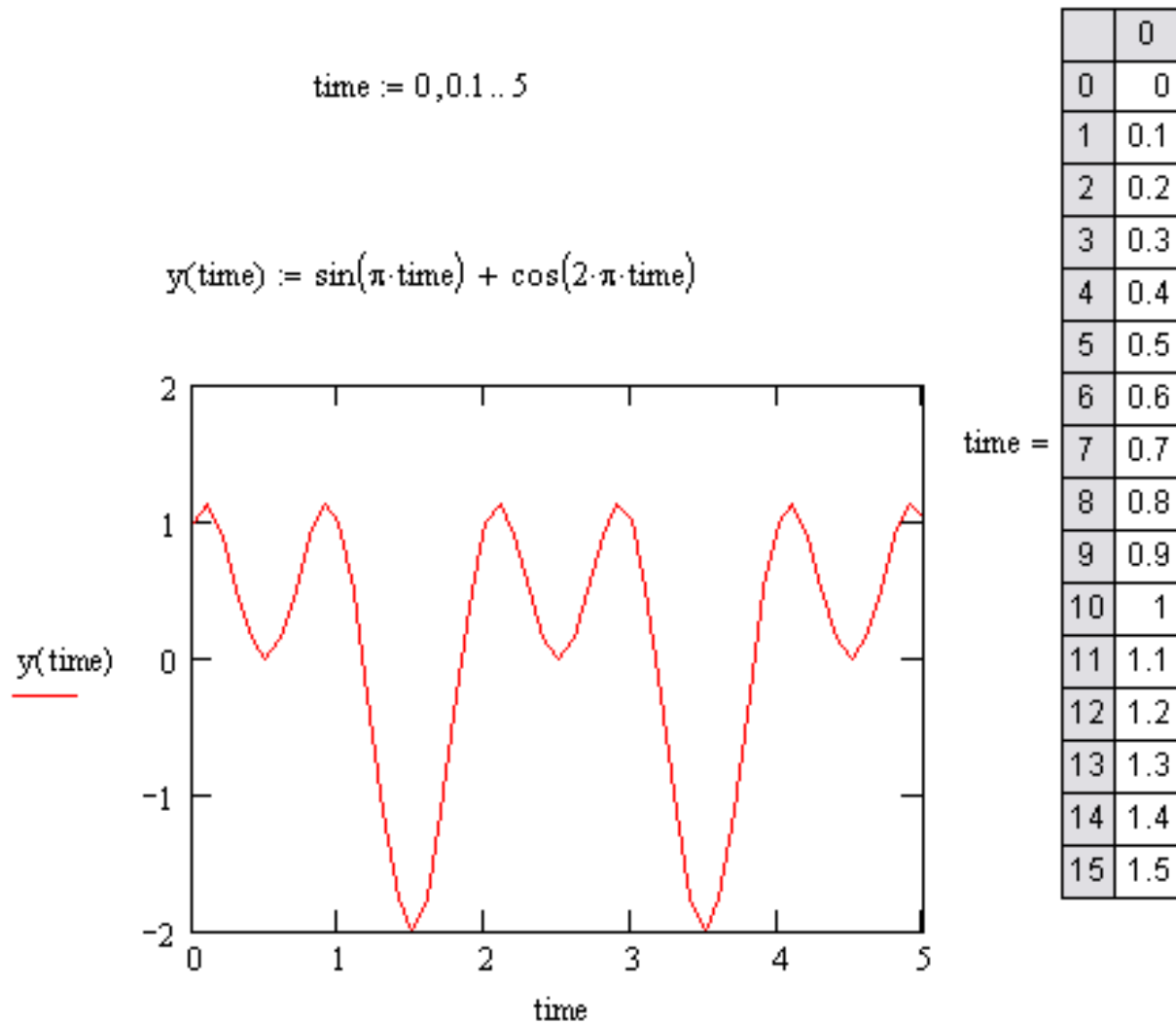
time=0-5 seconds (intervals of 0.1)

$y = \sin(\pi * \text{time}) + \cos(2 * \pi * \text{time})$

(Note: I have left out the commands to see if you remember HOW to enter them in)

Create a plot of y vs. time using indexed variables and functions

You should see the following:
This could be a signal or stray vibration?





MATHCAD 3-D plotting

There are two ways to plot a 3-D situation in MATHCAD

- By creating a matrix and specifically assigning the matrix to a surface or contour plot
- By creating a function of two variables and defining a range for the two variables.



MATHCAD 3-D plotting

Enter the following

```
[M][:][READPRN("c:\desktop\UCF\data4.txt")]
```

Then ***Insert*** graph and choose surface! Enter M for the variable at the bottom corner of the graph. CLICK on the graph and HOLD the mouse button. You can rotate the plot in 3-D space. You the scroll on the mouse to zoom in and out.

Double click on the graph!



Editing the 3-D graph

With the editing box open:

Choose the AXES TAB

- Notice X-Y-Z tabs. Change the Axes Color to whatever and the Turn the grid lines on and color them as well
- Under Appearance make sure “fill contours” is checked
- Click on TITLE tab and notice what is available.
- Choose “backpanes” choose fill backplane for each plane pair as well as the DRAW lines and color
- For kicks go to ADVANCED and enable FOG as well as view the various COLORMAPS

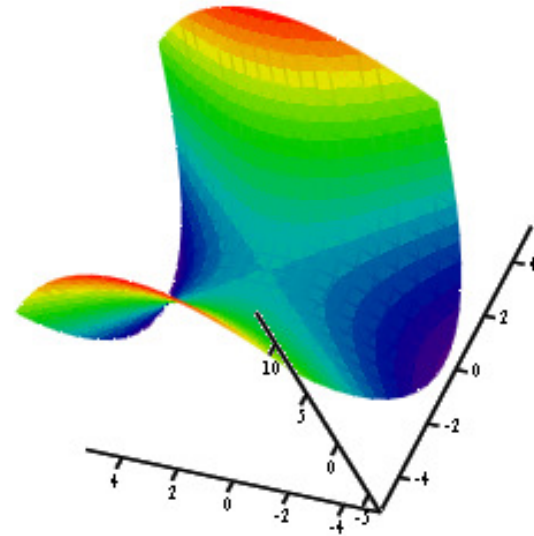
MATHCAD 3-D plotting

Enter the following:

$$z(x, y) := \frac{x^2}{2} - \frac{y^2}{4}$$

This is an equation for a hyperboloid!

Insert a surface plot then look at the contour plot which is the hyperbola as viewed from above





Complete the following:
