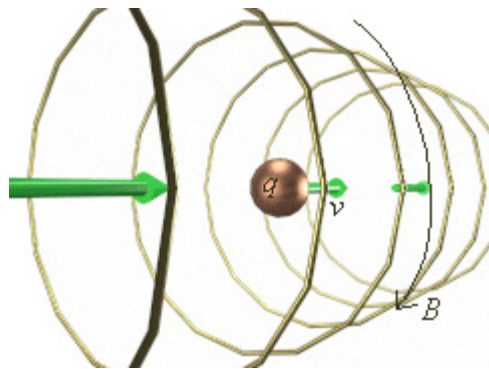
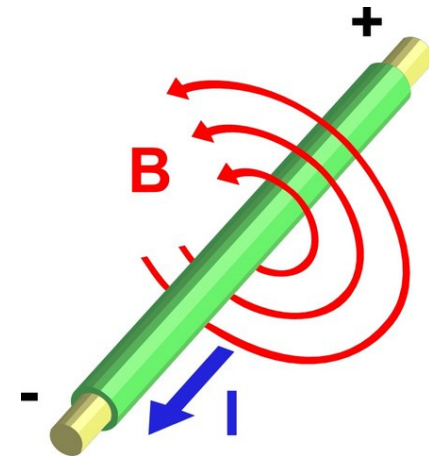

Magnetic Sources

AP Physics C

Sources of Magnetic Fields

In the last section, we learned that if a charged particle is moving and then placed in an **EXTERNAL** magnetic field, it will be acted upon by a magnetic force. The same is true for a current carrying wire.

The reason the wire and/or particle was moved was because there was an **INTERNAL** magnetic field acting around it. It is the interaction between these 2 fields which cause the force.



Can we define this **INTERNAL** magnetic field mathematically?

Biot-Savart Law (particles)

The magnetic field surrounding a moving charge can be understood by looking at the ELECTRIC FIELD of a point charge.

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}, \quad \frac{1}{4\pi\epsilon_0} = \text{constant}$$
$$B = \frac{\mu_0}{4\pi} \frac{qv}{r^2} \hat{r}, \quad \frac{\mu_0}{4\pi} = \text{constant}, \quad \hat{r} = \sin \theta$$

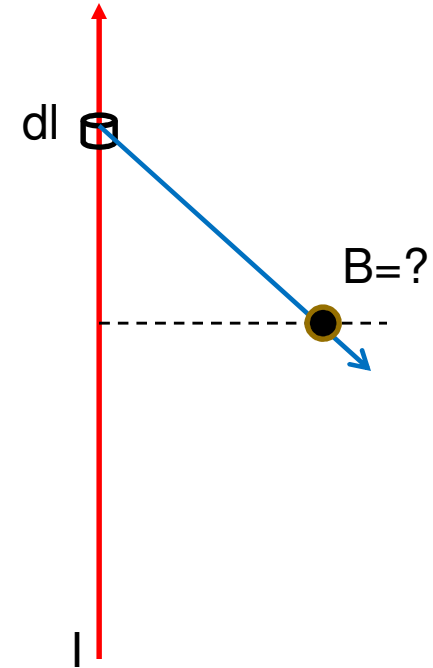
Here we see that the FIELD is directly related to the CHARGE and inversely related to the square of the displacement. The only difference in the case of the B-Field is that particle MUST be moving and the vectors MUST be perpendicular.

Biot-Savart Law (wires)

$$\mathbf{B} = \frac{\mu_0}{4\pi} \frac{dq\mathbf{v}}{r^2} \hat{\mathbf{r}}, \quad \frac{\mu_0}{4\pi} = \text{constant}, \quad \hat{\mathbf{r}} = \sin \theta$$

$$\mathbf{v} = \frac{dl}{dt} \rightarrow \mathbf{B} = \frac{\mu_0}{4\pi} \frac{dqdl}{dtr^2} \hat{\mathbf{r}}, \quad I = \frac{dq}{dt}$$

$$\mathbf{B} = \frac{\mu_0}{4\pi} \frac{Idl}{r^2} \sin \theta$$



This is for a current carrying element. The “dl” could represent a small amount of a wire. To find the ENTIRE magnetic field magnitude at a point away from the wire we would need to integrate over the length.

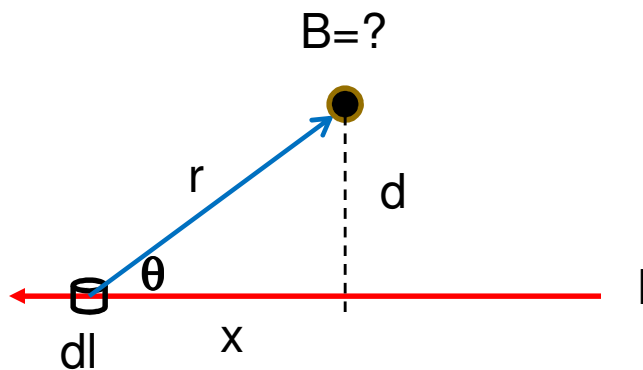
Biot-Savart Law (wires)

$$dB = \frac{\mu_0}{4\pi} \frac{Idl}{r^2} \sin \theta \quad \sin \theta = \frac{d}{\sqrt{d^2 + x^2}} \quad r = \sqrt{d^2 + x^2}$$

$$dB = \frac{\mu_0}{4\pi} \frac{Idl}{(d^2 + x^2)} \left(\frac{d}{\sqrt{d^2 + x^2}} \right) \rightarrow \frac{\mu_0}{4\pi} \frac{Idl d}{(d^2 + x^2)^{3/2}}$$

$$B = \int dB = \frac{\mu_0}{4\pi} Id \int \frac{dl}{(d^2 + x^2)^{3/2}}$$

Suppose we have a current carrying wire. A small current element of length “dl” is a distance “r” from a point directly above the wire at a distance “d”.

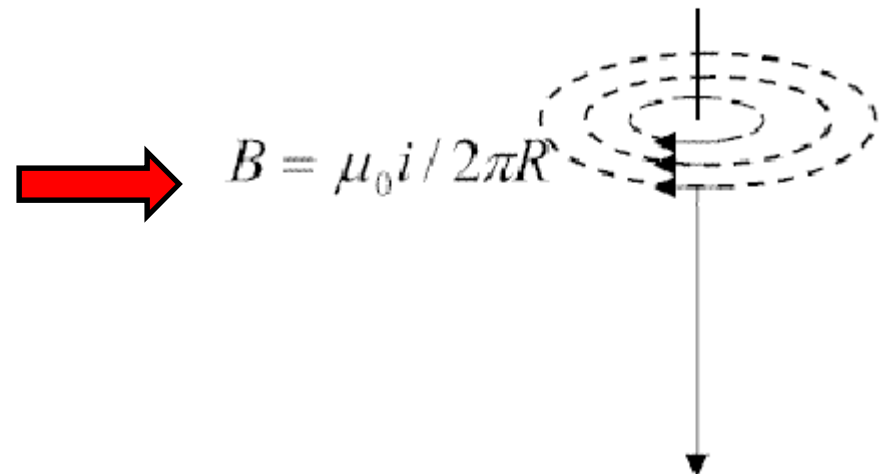


What is the magnetic field of ALL the current elements if the wire is straight and infinitely long?

Biot-Savart Law (wires)

$$B = \int dB = \frac{\mu_0 I d}{4\pi} \int_{-\infty}^{\infty} \frac{dl}{(d^2 + x^2)^{3/2}} =$$

$$B = \frac{\mu_0 I d 2}{4\pi d^2} = \frac{\mu_0 I}{2\pi d}$$

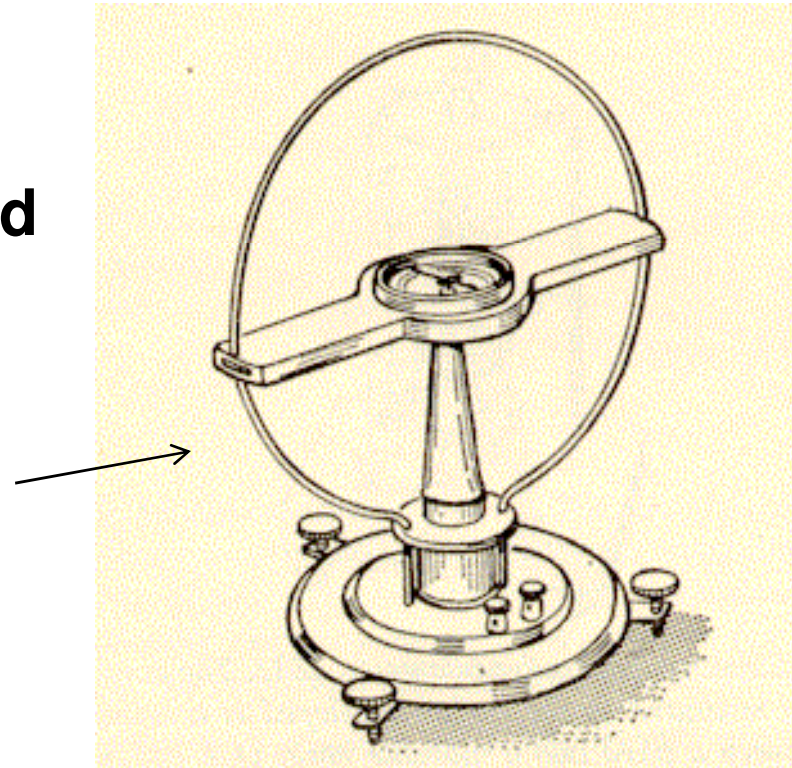


The result is the same equation we learned in the previous section. However, we **MUST** realize that this is only for a wire that is straight and infinitely long.

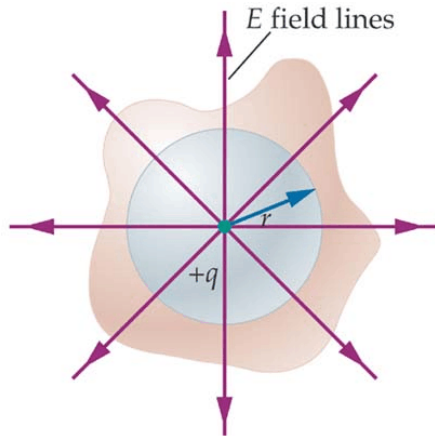
Biot-Savart Law (wires)

What is the equation for the magnitude of the magnetic field at the center of a current carrying loop?

Knowing this can be used in conjunction with a tangent galvanometer to solve for the magnetic field of Earth.



Ampere's Law & Gauss' Law



$$E \oint da = \frac{q_{enc}}{\epsilon_0}$$

$$B \oint da = 0$$

If you ENCLOSE a magnet, the # of field lines entering is EQUAL to the # of lines leaving, thus Gauss' Law sets it equal to ZERO unlike the case for an electric field.

Is there a way OTHER THAN the Biot-Savart Law to evaluate the magnetic field of a current carrying element?

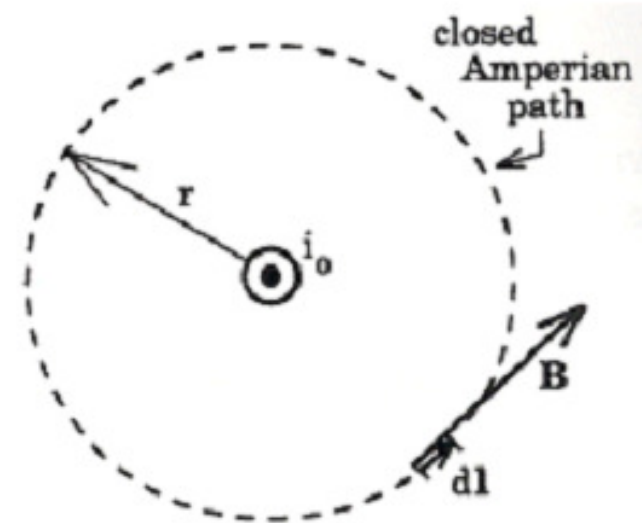
YES

Much like Gauss' Law, there is a way to determine the magnetic field of a current carrying element in a situation involving symmetry. The only thing that has to change is the PATH OF INTEGRATION and WHAT you are enclosing.

Ampere's Law

When using Gauss' Law we used the FLUX, which had the ELECTRIC, FIELD, E, and the AREA, A, parallel. This also enclosed the charge.

What 2 variables are parallel in this case and what are we enclosing?



The MAGNETIC FIELD, B, and the actual PATH LENGTH, L, that the magnetic field travels around the wire are parallel. It is the wire's CURRENT which is enclosed.

$$B \oint dl \propto I_{enc}$$

$$B \oint dl = \mu_0 I_{enc}$$

The magnetic field and the PATH of the field are both directly related to the current.

Ampere's Law

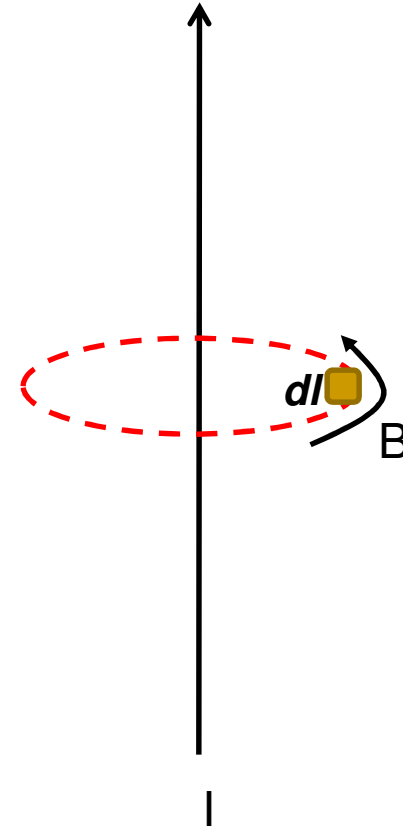
$$B \oint dl = \mu_0 I_{enc}$$

$$B(2\pi r) = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$

When we SUM all of the current carrying elements around the PATH of a circle we get the circle's circumference.

Once again we see we get the equation for the magnetic field around a long straight wire.



Example

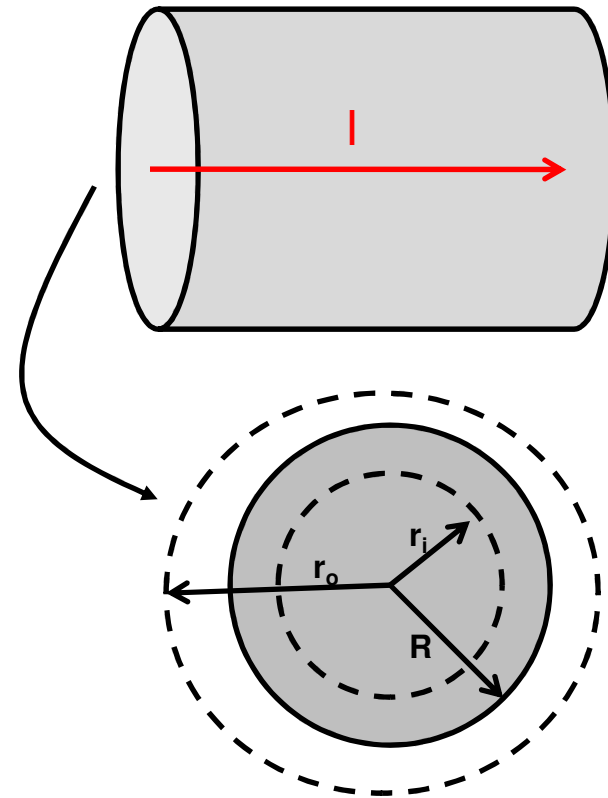
A long straight wire of radius R carries a current I that is uniformly distributed over the circular cross section of the wire. Find the magnetic field both outside the wire and inside the wire.

Let's look at the OUTSIDE field, $r_o > R$

$$B \oint dl = \mu_0 I_{enc}$$

$$B(2\pi r) = \mu_0 I$$

$$B_{outside} = \frac{\mu_0 I}{2\pi r}$$



Example

A long straight wire of radius R carries a current I that is uniformly distributed over the circular cross section of the wire. Find the magnetic field both outside the wire and inside the wire.

Let's look at the INSIDE field, $r_i < R$

We first need to identify exactly what is the ENCLOSED current. It isn't, "I", but rather a FRACTION of "I".

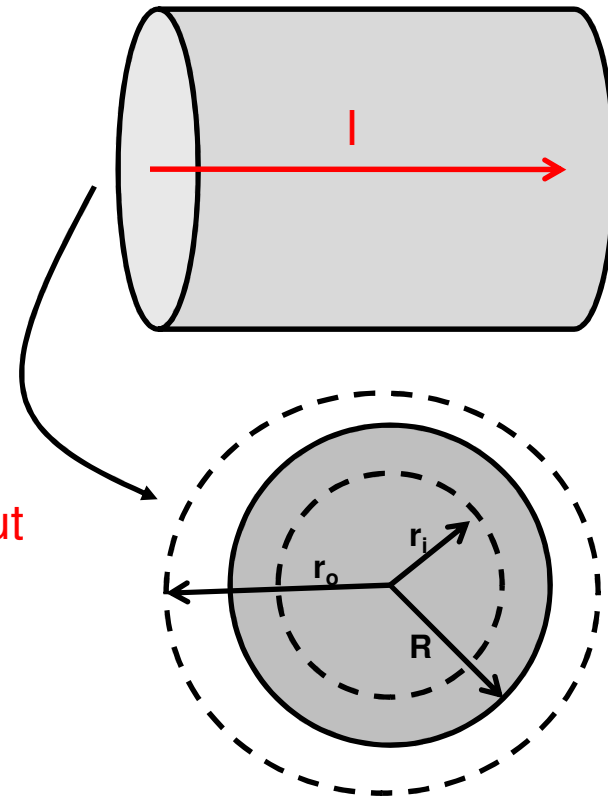
$$\frac{I_{enc}}{\pi r^2} = \frac{I}{\pi R^2} \quad I_{enc} = I \frac{r^2}{R^2}$$

$$B \oint dl = \mu_0 I_{enc}$$

$$B(2\pi r) = \mu_0 I \frac{r^2}{R^2}$$

$$B_{inside} = \frac{\mu_0 I r}{2\pi R^2}$$

Since the current is distributed throughout the cross section we can set up a ratio of the currents as it relates to the cross sectional area.

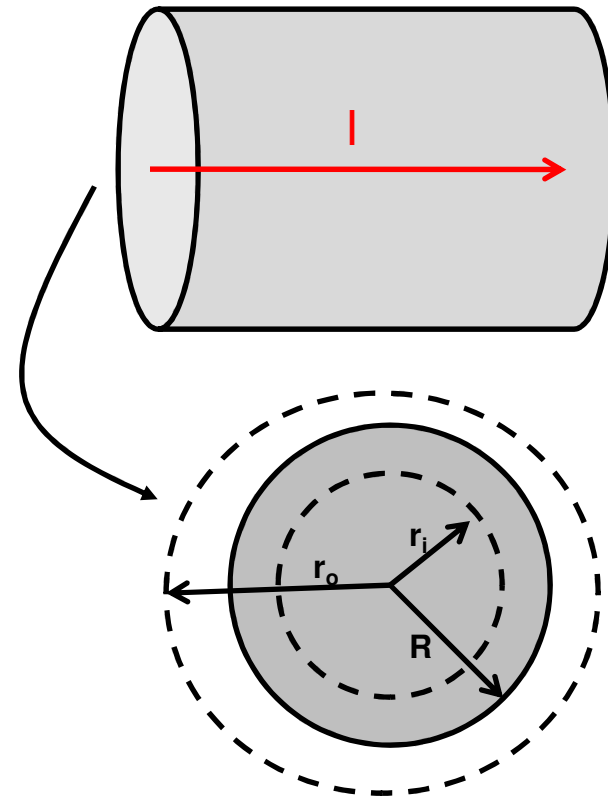
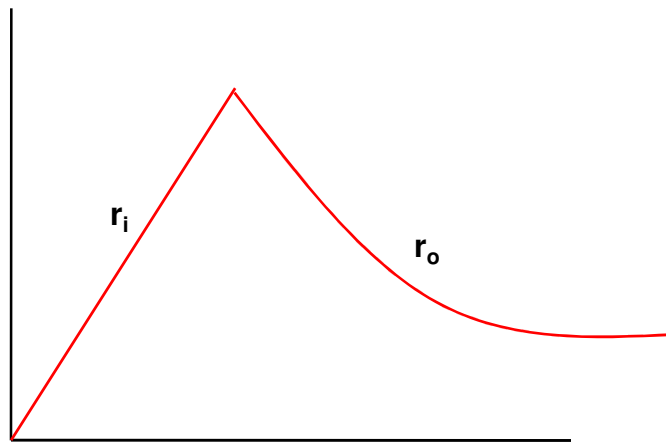


Example

How could the magnetic field be graphically displayed?

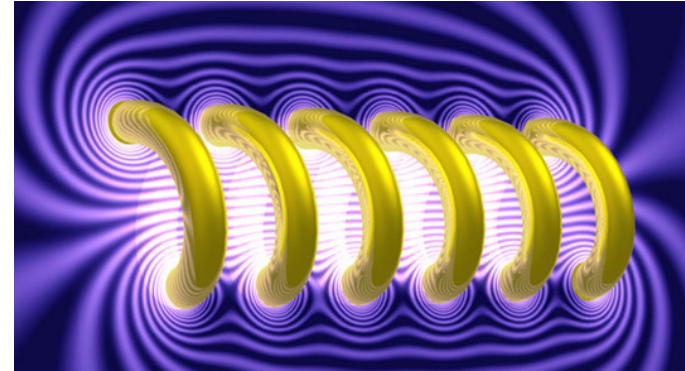
$$B_{outside} = \frac{\mu_0 I}{2\pi r} \quad B \propto \frac{1}{r}$$

$$B_{inside} = \frac{\mu_0 I r}{2\pi R^2} \quad B \propto r$$



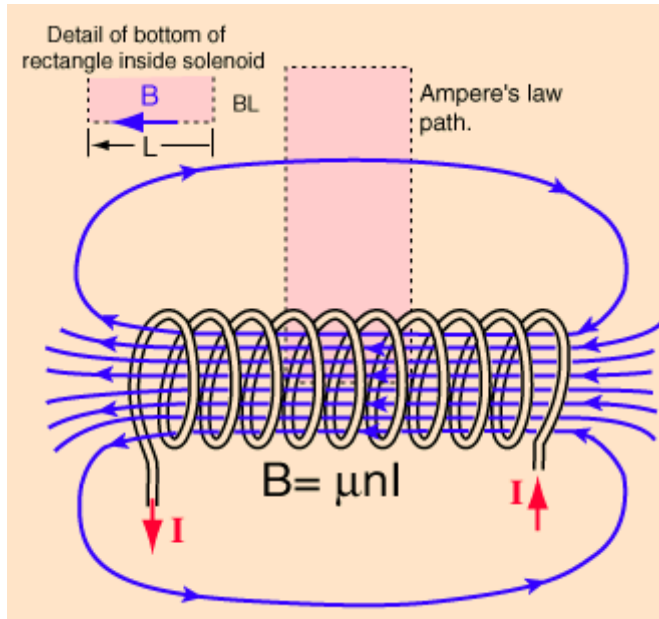
Applications of Ampere's Law – A Solenoid

A solenoid is basically a bunch of loops of wire that are tightly wound. It is analogous to a capacitor which can produce a strong electric field. In this case it can produce a strong MAGNETIC FIELD.



Solenoids are important in engineering as they can convert electromagnetic energy into linear motion. All automobiles use what is called a “starter solenoid”. Inside this starter is a piston which is pushed out after receiving a small amount of current from the car’s battery. This piston then completes a circuit between the car’s battery and starter motor allowing the car to operate.

Applications of Ampere's Law – A Solenoid



It is important to understand that when you enclose a certain amount of turns that the magnetic field runs through the center of the solenoid. As a result the field lines and the length of the solenoid are parallel. This is a requirement for Ampere's Law.

The first thing you must understand is what is the enclosed current. It is basically the current, I , times the # of turns you enclose, N .

When you integrate all of the small current elements they ADD up to the length of the solenoid, L

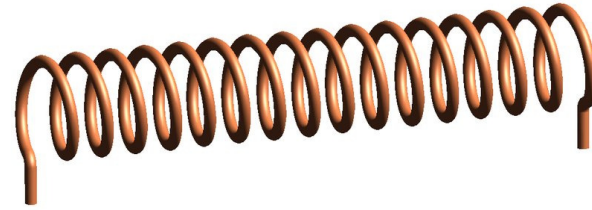
$$\oint B \cdot dl = \mu_0 I_{enc}$$

$$B(L) = \mu_0 (NI)$$

$$B = \mu_0 \frac{N}{L} I, \quad n = \# \text{ turns per length} = \frac{N}{L}$$

$$B_{solenoid} = \mu_0 n i$$

Example



A solenoid has a length $L = 1.23$ m and an inner diameter $d = 3.55$ cm, and it carries a current of 5.57 A. It consists of 5 close-packed layers, each with 850 turns along length L . What is the magnetic field at the center?

$$n = \# \text{ turns per length} = \frac{N}{L}$$

$$B_{\text{solenoid}} = \mu_0 n i$$

$$B = (1.26 \times 10^{-6}) 5 \times \frac{850}{1.23} (5.57)$$

$$B = \mathbf{0.024 \text{ T}}$$