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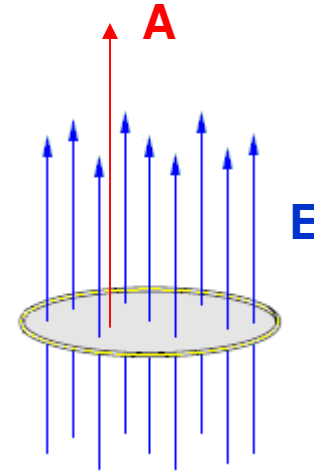
# Gauss' Law

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AP Physics C

# Electric Flux

Let's start by defining an area on the surface of an object. The magnitude is "A" and the direction is directed perpendicular to the area like a force normal.



Flux ( or FLOW) is a general term associated with a FIELD that is bound by a certain AREA. So **ELECTRIC FLUX** is any **AREA** that has a **ELECTRIC FIELD** passing through it.

We generally define an AREA vector as one that is perpendicular to the surface of the material. Therefore, you can see in the figure that the AREA vector and the Electric Field vector are **PARALLEL**. This then produces a **DOT PRODUCT** between the 2 variables that then define flux.

# Electric Flux

The electric field lines look like lines of a "fluid". So you can imagine these lines are flowing (even though nothing is really flowing). **The word FLUX roughly means FLOW.**

So based on this idea we can define the **ELECTRIC FLUX** as the **ELECTRIC FEILD** through a **SURFACE AREA**.

Since the area vector is defined as perpendicular to the surface and the electric field goes through it, we define this equation as a dot product, similar to the **work** function.

$$\Phi_E = E \cdot A = EA \cos \theta$$

$$d\Phi = \oint E da$$

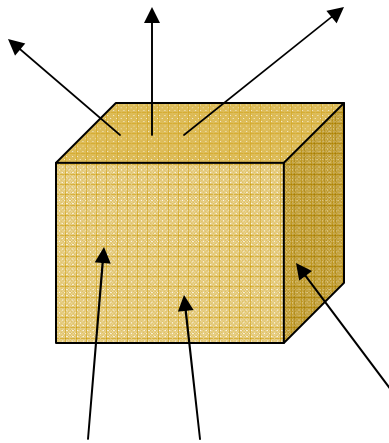
*A differential amount of flux is the cross product between the electric field and a differential amount of area.*

Since you want the total flux, you integrate to sum up all the small areas. Thus the **TOTAL FLUX** is found by integrating over the **ENTIRE SURFACE**. ***The circle on the integration sign simply means the surface is CLOSED!!.***



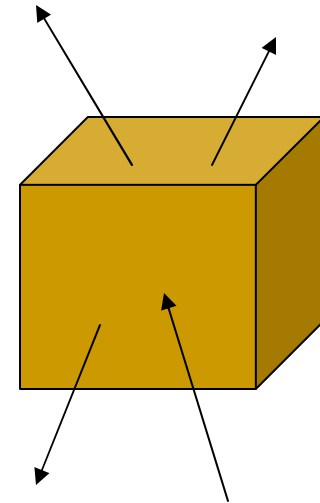
# Electric Flux

Visually we can try to understand that the flux is simply the # of electric field lines passing through any given area.



In the left figure, the flux is zero.

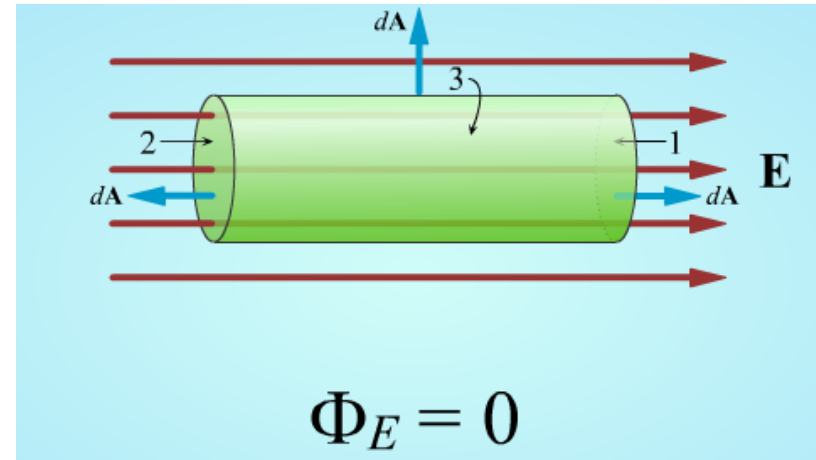
In the right figure, the flux is 2.



- When E lines pass outward through a closed surface, the FLUX is positive
- When E lines go into a closed surface, the FLUX is negative

# Electric Flux

What is the electric flux of this cylinder?



$$\Phi_E = \sum \Phi = \Phi_1 + \Phi_2 + \Phi_3, \quad E = \text{constant}, \quad A_1 = A_2$$

$$\Phi_E = EA_1 \cos 0 + EA_2 \cos 180 + EA_3 \cos 90$$

$$\Phi_E = EA_1(1) + EA_2(-1) + 0$$

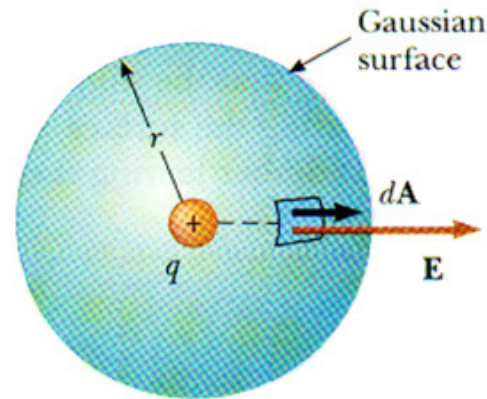
$$\Phi_E = 0 \quad \text{What does this tell us?}$$

This tells us that there are NO sources or sinks INSIDE the cylindrical object.

# Gauss' Law

Where does a fluid come from? A spring! The spring is the SOURCE of the flow.

Suppose you enclose the spring with a closed surface such as a sphere. If your water accumulates within the sphere, you can see that the total flow out of the sphere is equal to the rate at which the source is producing water.



In the case of electric fields the source of the field is the CHARGE!

So we can now say that the **SUM OF THE SOURCES WITHIN A CLOSED SURFACE IS EQUAL TO THE TOTAL FLUX THROUGH THE SURFACE.** This has become known as Gauss' Law

# Gauss' Law

$$q_{\text{enclosed}} \propto \Phi$$

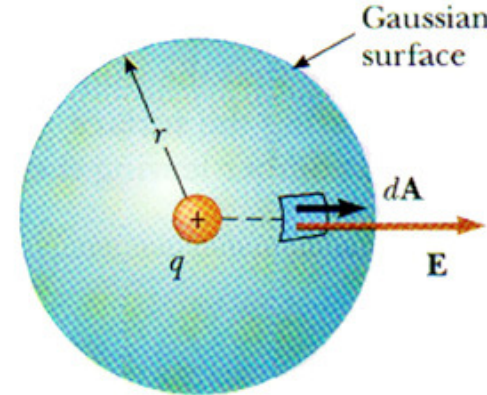
$\epsilon = \text{Constant of Proportionality}$

$$q_{\text{enclosed}} = \epsilon \Phi$$

$$\Phi = \frac{q_{\text{enc}}}{\epsilon} = \oint E \cdot dA$$

The vacuum permittivity constant is the constant of proportionality in this case as the flow can be interrupted should some type of material come between the flux and the surface area. Gauss' Law then is derived mathematically using 2 known expressions for flux.

The electric flux (flow) is in direct proportion to the charge that is enclosed within some type of surface, which we call Gaussian.



$$\oint E \cdot dA = \frac{q_{\text{enc}}}{\epsilon_0}$$

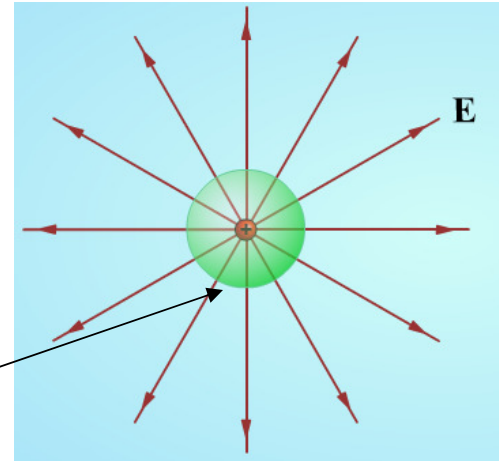
# Gauss' Law – How does it work?

Consider a POSITIVE POINT CHARGE,  $Q$ .

**Step 1** – Is there a source of symmetry?

*Yes, it is spherical symmetry!*

You then draw a shape in such a way as to obey the symmetry and **ENCLOSE** the charge. In this case, we enclose the charge within a sphere. This surface is called a **GAUSSIAN SURFACE**.



**Step 2** – What do you know about the electric field at all points on this surface? *It is constant.*

$$E \oint da = \frac{q_{enc}}{\epsilon_0}$$

The “E” is then brought out of the integral.

# Gauss' Law – How does it work?

**Step 3** – Identify the area of the Gaussian surface?

*In this case, summing each and every  $dA$  gives us the surface area of a sphere.*

$$E(4\pi r^2) = \frac{q_{enc}}{\epsilon_0}$$

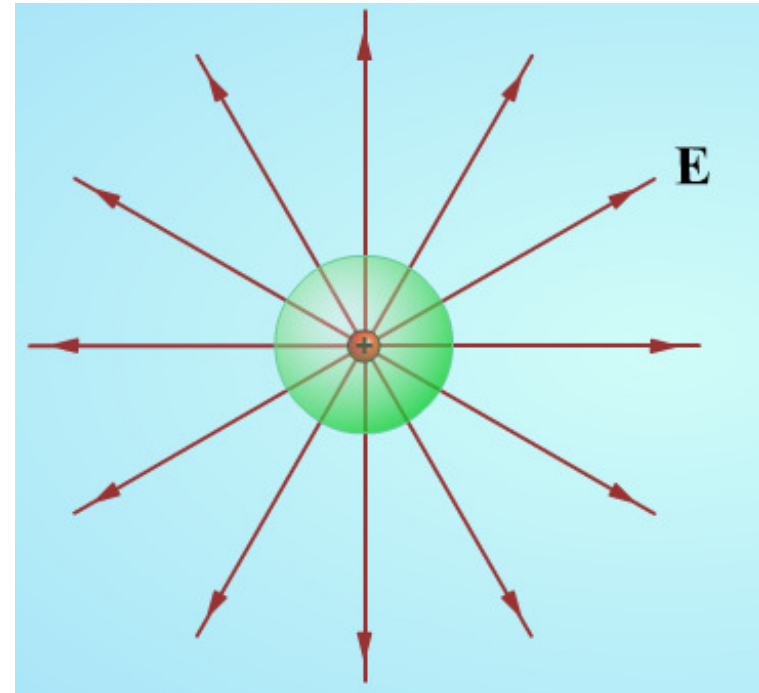
**Step 4** – Identify the charge enclosed?

*The charge enclosed is  $Q$ !*

$$E(4\pi r^2) = \frac{Q}{\epsilon_0} \rightarrow$$

$$E = \frac{Q}{4\pi r^2 \epsilon_0}$$

**This is the equation for a POINT CHARGE!**



# Gauss & Michael Faraday

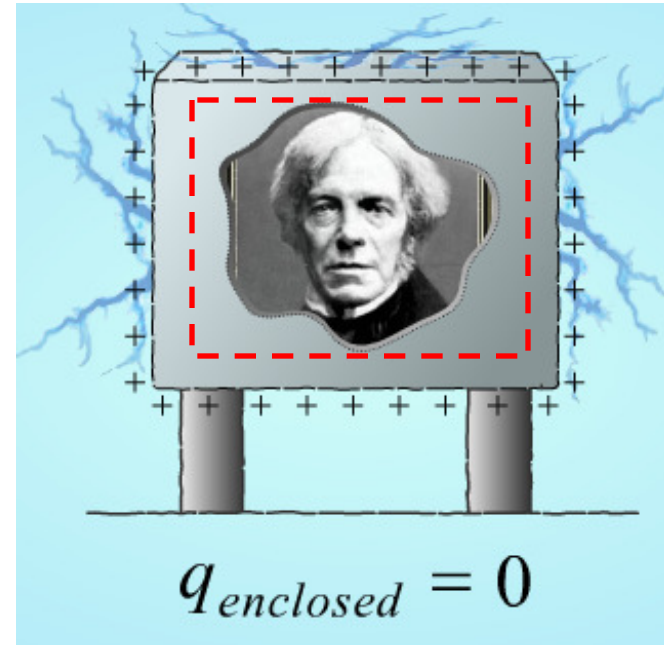
Faraday was interested in how charges move when placed inside of a conductor. He placed a charge inside, but as a result the charges moved to the outside surface.

Then he choose his Gaussian surface to be just inside the box.

$$\oint E \cdot da = \frac{q_{enc}}{\epsilon_o} \rightarrow 0(A) = \frac{q_{enc}}{\epsilon_o}$$

$$q_{enc} = 0$$

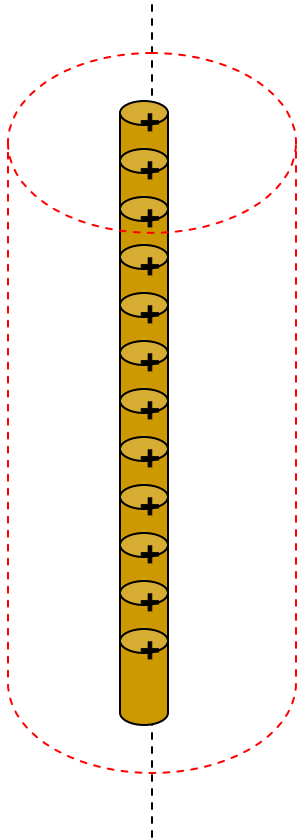
He verified all of this because he DID NOT get shocked while INSIDE the box. This is called Faraday's cage.



# Gauss' Law and cylindrical symmetry

Consider a line( or rod) of charge that is very long (infinite)

We can **ENCLOSE** it within a **CYLINDER**. Thus our Gaussian surface is a cylinder.



$$E \oint da = \frac{q_{enc}}{\epsilon_o} \quad E(2\pi rL) = \frac{q_{enc}}{\epsilon_o}$$

$$E(2\pi rL) = \frac{\lambda L}{\epsilon_o}$$

$$E = \frac{\lambda}{2\pi r \epsilon_o}$$

$$RECALL: Macro \rightarrow \lambda = \frac{Q}{L}$$

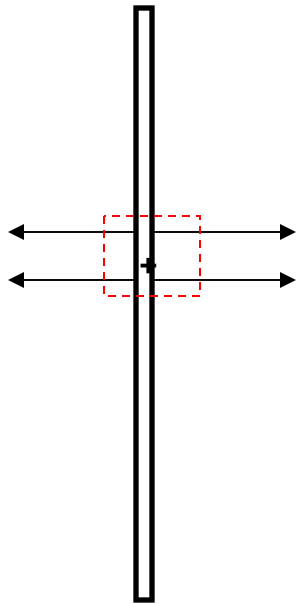
$$Q = \lambda L = q_{enc}$$

$$A_{cylinder} = 2\pi rL$$

This is the same equation we got doing extended charge distributions.

# Gauss' Law for insulating sheets and disks

A charge is distributed with a uniform charge density over an infinite plane **INSULATING** thin sheet. Determine **E** outside the sheet.



For an insulating sheet the charge resides **INSIDE** the sheet. Thus there is an electric field on **BOTH** sides of the plane.

$$\oint E \cdot dA = \frac{q_{enc}}{\epsilon_0}$$

$$EA + EA = \frac{Q}{\epsilon_0} \rightarrow 2EA = \frac{Q}{\epsilon_0}$$

$$\sigma = \frac{Q}{A}, \quad 2EA = \frac{\sigma A}{\epsilon_0}$$

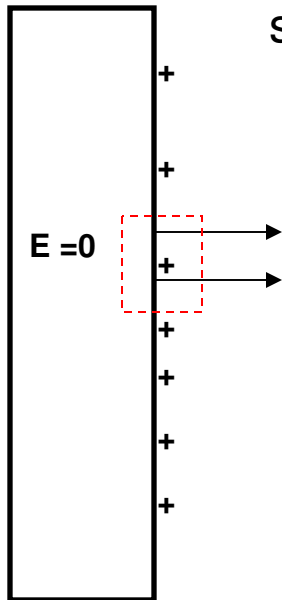
$$E = \frac{\sigma}{2\epsilon_0}$$

← This is the same equation we got doing extended charge distributions.

# Gauss' Law for conducting sheets and disks

A charge is distributed with a uniform charge density over an infinite thick conducting sheet. Determine E outside the sheet.

For a thick conducting sheet, the charge exists on the surface only



$$\oint E \cdot dA = \frac{q_{enc}}{\epsilon_0}$$

$$EA = \frac{Q}{\epsilon_0}$$

$$\sigma = \frac{Q}{A}, \quad EA = \frac{\sigma A}{\epsilon_0}$$

$$E = \frac{\sigma}{\epsilon_0}$$

# In summary

Whether you use electric charge distributions or Gauss' Law you get the SAME electric field functions for symmetrical situations.

$$E = \frac{Q}{4\pi\epsilon_0 r^2} \rightarrow dE = \frac{dq}{4\pi\epsilon_0 r^2}$$
$$\oint E \cdot dA = \frac{q_{enc}}{\epsilon_0}$$

Function	Point, hoop, or Sphere (Volume)	Disk or Sheet (AREA) “insulating and thin”	Line, rod, or cylinder (LINEAR)
Equation	$E = \frac{Q}{4\pi\epsilon_0 r^2}$	$E = \frac{\sigma}{2\epsilon_0}$	$E = \frac{\lambda}{2\pi\epsilon_0 r}$